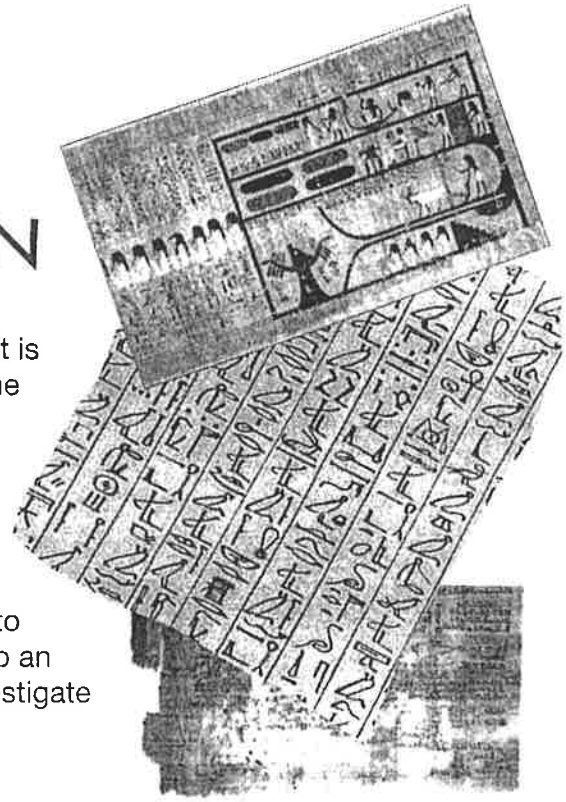
Student Explorations in Mathematics



DIVIDE LIKE AN EGYPTIAN

Imagine that you put your math notebook into a time capsule and it is recovered 5000 years from now. Suppose that your notebook is one of the only existing written records from the twenty-first century. Future civilizations could learn about our understanding of mathematics from your work.

A rare written record from the ancient Egyptians shows calculations and answers to questions about division and what we commonly think of as fractions. This activity will introduce you to the notation they used and allow you to reproduce their work. Keep an open mind as you answer questions, taking the opportunity to investigate what might be a familiar topic in a new way.

Figure 1. The Rhind Papyrus Illustrated the Egyptian Method of Multiplication.

Source: http://www-history.mcs.st-and.ac.uk/HistTopics/Egyptian_papyri.html

Writing Numbers

About 5000 years ago, the ancient Egyptians used hieroglyphs to represent numbers (see **table 1**).

Table 1. Egyptian Number Symbols

1	10	100	1000
𓏺	𓎆	𓍢	𓆼
Single Stroke	Cattle Hobble	Rope Coil	Lotus Plant

1. What number does each hieroglyph below represent?

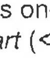
a. 
b. 
c. 

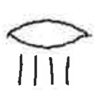
2. Draw the hieroglyph for each number below.

   a. 8

   b. 52

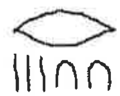
   c. 1231

Egyptians dealt with questions of division when sharing resources. For example, if one loaf of bread is shared among four people, what part of the loaf does each person receive? This question is represented in modern mathematics as 1 ÷ 4. We might solve this with the decimal value 0.25; or we might say each person receives one-fourth of a loaf of bread. The Egyptians would show that each person receives one of four equal parts by putting the symbol for *part* () above the symbol for *four*, as shown below:



3. Draw the hieroglyph that shows one of three equal parts.

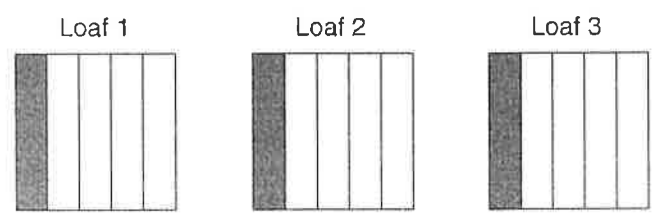
4. What does the following hieroglyph show?



Questions 3 and 4 represent the answers to the division questions 1 ÷ 3 and 1 ÷ 23, respectively. Unfortunately, the Egyptians did not have a symbol for two parts or three parts, only for one part. They could represent one of five equal parts, but not three of five equal parts. (You may have some thoughts about how to represent that idea, but the Egyptians did not have the benefit of your insight.) To represent the answer to such a question as 3 ÷ 5, they had to do some mathematical computation.

Dividing with Pictures

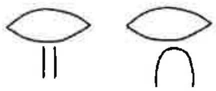
Consider sharing three loaves of bread equally among five people, or the division expression 3 ÷ 5. We can share equally by dividing each loaf into five pieces and giving every person one piece from each loaf. Every person receives three pieces that are each one-fifth of a loaf, illustrated below by the shaded portions.



You might think the Egyptians would represent the share one person receives with the hieroglyph below, one symbol for each piece.

Although they wrote symbols next to one another to represent addition, they did not allow the same symbol to repeat within the representation of one quantity. They would use a different representation and therefore need a different method. Their answer to 3 ÷ 5 is on the next page (in question 5).

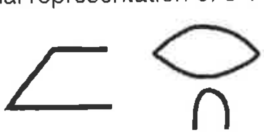
5. What does the hieroglyph below represent?



6. Determine a process that the Egyptians could use to distribute three loaves among five people equally.

Egyptians actually had a different symbol for one-half:

So, their final representation of 3 ÷ 5 looked like this:



7. Use the same method to show an Egyptian solution to 4 ÷ 5, sharing four loaves equally among five people.

8. Use this method to show the Egyptian solution to 3 ÷ 8.

The Modern Notation for Ancient Ideas

You may have already concluded that the Egyptians were actually dealing with fractions. The question of sharing three loaves among five people can also be represented with the modern notation $\frac{3}{5}$. For the most part, Egyptian symbols used only *unit* fractions (a fraction with the number 1 as the numerator). Other fractions were represented without repeating the same fractions by using sums of progressively smaller unit fractions. For example, when sharing the three loaves among five people, each person receives one-half loaf and one-tenth loaf. The modern notation would be $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$. We will call this representation an *Egyptian fraction*. From here on, we will use this modern notation for the Egyptian fractions.

9. Emily found that the Egyptian fraction for $\frac{7}{12}$ is $\frac{1}{2} + \frac{1}{12}$. How might she have determined her answer?

10. Determine an Egyptian fraction for $\frac{3}{8}$.

In addition to the  symbol for one-half, the Egyptians had other symbols for a few fractions that were not unit fractions. One of those was $\frac{2}{3}$, represented as . We can now expand our definition of an Egyptian fraction to be written as the sum of different fractions where each one is either a unit fraction or $\frac{2}{3}$.

11. Determine an Egyptian fraction for $\frac{5}{6}$. Does your answer include $\frac{2}{3}$? Why, or why not?

12. Determine an Egyptian fraction for $\frac{3}{4}$. This can be done in more than one way. Find at least two ways.

Comparing Fractions

One way to compare fractions is to find common denominators. However, there are clever methods that do not require a common denominator. For example, we know that a fraction is equal to $\frac{1}{2}$ if the numerator is exactly one-half of the denominator, such as $\frac{4}{8}$ or $\frac{11.5}{23}$. The fraction $\frac{3}{8}$ is less than $\frac{1}{2}$ because the numerator (3) is less than one-half the denominator (half of 8 is 4).

13. Explain why $\frac{5}{9}$ is greater than $\frac{1}{2}$.

14. Decide which is smaller, $\frac{9}{14}$ or $\frac{5}{12}$, by comparing each fraction to $\frac{1}{2}$.

Consider the unit fractions $\frac{1}{2}$ and $\frac{1}{5}$. If we divide one whole loaf into two equal pieces, those pieces will be larger than the same size loaf divided into five equal pieces. Since we are comparing only one piece in each case, we know that a one-half-size piece is larger than a one-fifth-size piece. If we share one loaf equally between two people, each person gets a larger piece than if we share one loaf equally among five people. So $\frac{1}{2}$ is larger than $\frac{1}{5}$.

15. Demonstrate which is larger, $\frac{1}{4}$ or $\frac{1}{3}$. Explain how the strategy can be applied to all unit fractions.

16. How can you compare $\frac{4}{9}$ and $\frac{4}{11}$?

17. Now imagine two pizzas of the same size. One has $\frac{7}{8}$ remaining, and the other has $\frac{11}{12}$ remaining. How can this strategy help us determine which pizza has more remaining? (Hint: Consider how much of each pizza is missing.)

18. Use the strategy to compare $\frac{11}{13}$ and $\frac{17}{19}$. Explain your reasoning as to which is smaller.

We can also compare fractions by looking at their Egyptian form. Consider the Egyptian fractions below:

$$\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$$

$$\frac{5}{8} = \frac{1}{2} + \frac{1}{8}$$

Both fractions are greater than $\frac{1}{2}$, so we can compare the second parts. Since $\frac{1}{8}$ is greater than $\frac{1}{10}$, we can conclude that $\frac{5}{8}$ is greater than $\frac{3}{5}$.

19. Use Egyptian fractions to compare $\frac{8}{15}$ and $\frac{6}{11}$. Explain your reasoning as to which is smaller.

20. When ordering a list of fractions, it is often helpful to apply several comparison strategies. Order the following list of fractions from least to greatest, describing the strategies you use:

$\frac{11}{13}, \frac{3}{5}, \frac{3}{7}, \frac{8}{14}, \frac{9}{16}, \frac{8}{10}$

Can you ...

- compare $\frac{22}{23}$ and $\frac{26}{27}$?
- determine the unit fraction sum for $\frac{2}{29}$?
- find a sum for $\frac{7}{8}$ with three different unit fractions?
- find number systems that write their numbers differently than the Egyptian or Hindu-Arabic systems?

Did you know that ...

- the Rhind Papyrus, now located in The British Museum, was named after the Scotsman Henry Rhind, who purchased it in 1858?
- Fibonacci proved that every simple fraction (a fraction where the numerator and denominator are both integers) can be represented as the sum of unit fractions? The method for finding the sum is called a *greedy algorithm*.
- an infinite number of unit fraction sum representations exist for every simple fraction (where the numerator and denominator are integers)?
- unit fractions are used in the resistance formula for parallel circuits?

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